

## Warm-Up

Simplify.

$$a.) \frac{9x^2 + 81x}{x^3 + 8x^2 - 9x}$$

$$\frac{9x(x+9)}{x(x^2+8x-9)} = \frac{9x(x+9)}{x(x+9)(x-1)}$$

$$\frac{\cancel{9} \cancel{x} \cancel{-9}}{\cancel{x} \cancel{-1}}$$

$$= \boxed{\frac{9}{x-1}}$$

$$b.) \frac{3r^2 - 39r + 90}{r^2 - 3r - 70}$$

$$\frac{3(r^2 - 13r + 30)}{(r-10)(r+7)} \quad \begin{matrix} \cancel{30} \\ \cancel{-10} \quad \cancel{-3} \\ \cancel{-13} \end{matrix}$$

$$\frac{3(\cancel{r-10})(r-3)}{(\cancel{r-10})(r+7)}$$

$$\boxed{\frac{3(r-3)}{r+7}}$$

## 2C.2 : Vertical + Horizontal Asymptotes

### Rules for Asymptotes

$$f(x) = \frac{N(x)}{D(x)} \quad \begin{matrix} N + D \text{ are polynomials} \\ \text{degree of } N \text{ is } n \\ \text{degree of } D \text{ is } m \end{matrix}$$

### Vertical Asymptotes (VA)

where  $D(x) = 0$

### Horizontal Asymptote (HA)

Compare the degrees of  $\begin{matrix} N(x) & + & D(x) \\ n & & m \end{matrix}$

a.)  $n < m \Rightarrow$  HA is  $\boxed{y=0}$

b.)  $n = m \Rightarrow$  HA is ratio of lead coefficients  $\boxed{y = \frac{a}{b}}$

for example:  $f(x) = \frac{4x^3 + 2x^2 - x + 9}{6x^3 + 8}$   $n = m$   
 $HA \Rightarrow y = \frac{4}{6}$

c.)  $n > m$   $\boxed{\text{NO}}$  HA

EX1) Find the domain, horizontal and vertical asymptotes of  $f$ .

$$f(x) = \frac{2x+1}{x^2-4}$$

$$\begin{aligned} x^2-4 &= 0 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

Domain: all  $x$ ,  $x \neq \pm 2$

VA

$$\begin{aligned} x^2-4 &= 0 \\ x^2 &= 4 \end{aligned}$$

$$x = \pm 2$$

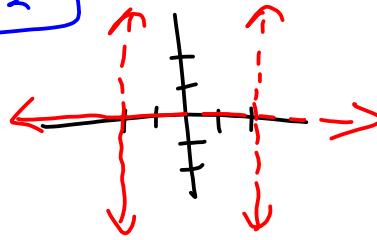
HA

$$n=1, m=2$$

$$n < m$$

$$HA \Rightarrow$$

$$y=0$$



$$f(x) = \frac{(2x+1)}{(x^2-4)}$$

EX2) Find any horizontal + vertical Asymptotes

$$f(x) = \frac{x+1}{x^2-1}$$

HA

$$n=1, m=2$$

$$n < m \quad HA \Rightarrow y=0$$

VA

$$f(x) = \frac{\cancel{x+1}}{\cancel{x+1}(x-1)}$$

$$x-1=0$$

$$x=1$$

VA

$$\cancel{x-1}$$

