

3.1 Exponential Functions + Their Graphs

Today you will evaluate exponential functions and graph them.

$$f(x) = a^x$$

exponential function

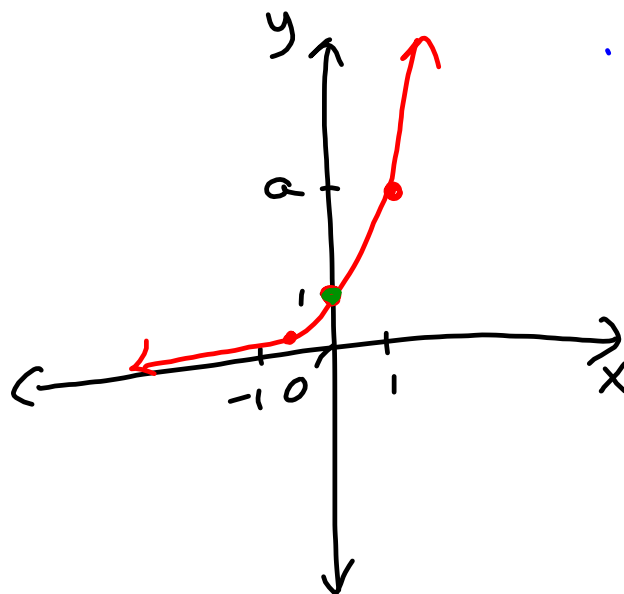
where $a > 0$, $a \neq 1$, $x \in \mathbb{R}$

$$y = 2^x$$

x	y
1	2
2	4
3	8

Graph of
 $y = a^x$

x	$y = a^x$	(x, y)
1	$a^1 = a$	$(1, a)$
0	$a^0 = 1$	$(0, 1)$
-1	$a^{-1} = \frac{1}{a}$	$(-1, \frac{1}{a})$



Reminder:

$$x^{-1} = \frac{1}{x^1}$$

$$3^{-1} = \frac{1}{3}$$

$$\left(\frac{1}{3}\right)^{-1} = \frac{1}{\left(\frac{1}{3}\right)^1} = 3$$

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

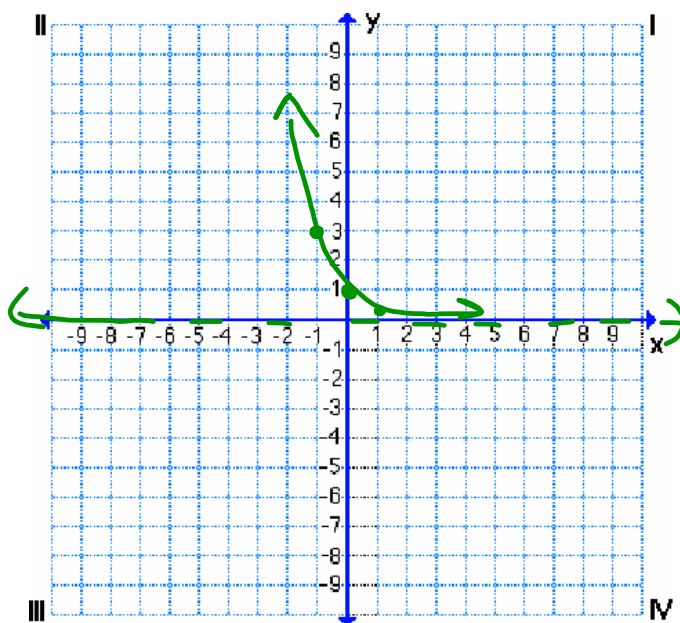
Reminder #2:

$$\left(\frac{1}{3}\right)^2 = \frac{1^2}{3^2} = \frac{1}{9}$$

$$\left(\frac{1}{3}\right)^{-2} = \frac{1^{-2}}{3^{-2}} = \left(\frac{1}{3^2}\right) = \left(\frac{1}{9}\right) = 9$$

Ex 2 Graph $y = \left(\frac{1}{3}\right)^x$

x	$y = \left(\frac{1}{3}\right)^x$	(x, y)
-1	$\left(\frac{1}{3}\right)^{-1} = 3$	(-1, 3)
0	$\left(\frac{1}{3}\right)^0 = 1$	(0, 1)
1	$\left(\frac{1}{3}\right)^1 = \frac{1}{3}$	$\left(1, \frac{1}{3}\right)$



Translations

$y = a^{x-h} + k$

h is horizontal shift left or right

k is vertical shift up or down

Steps for translation

1st step - graph the parent $y = a^x$

2nd step - then translate up/down left/right

#N3.1

1. $y = 3^{x-1} + 1$

right 1 up 1 $y = 3^x$

x	$y = 3^x$	(x, y)
-1	$\frac{1}{3}$	$(-1, \frac{1}{3})$
0	1	(0, 1)
1	3	(1, 3)

3.2 Logarithmic Functions + Their Graphs

Today you will evaluate
and graph logarithmic
functions

The diagram illustrates the relationship between logarithmic and exponential forms. On the left, the logarithmic equation $y = \log_a x$ is enclosed in a green box. A red arrow labeled "exponent" points to the y , and another red arrow labeled "base" points to the a . A blue double-headed arrow \Leftrightarrow connects this to the right. On the right, the exponential equation $a^y = x$ is enclosed in a green box. A red arrow labeled "exponent" points to the y , and another red arrow labeled "base" points to the a .

Using Calc:
 $x = \log_3 10$
 $x \approx 2.0959$

$3^x = 10$

By guess + check
 We found the answer

X Too Little:
 $3^2 = 9$
 $x = 2$

X Too Big:
 $3^3 = 27$
 $x = 3$

$3^{2.1} = 10.0451$
 $3^{2.09} = 9.9353$
 $3^{2.095} = 9.9909$
 $3^{2.096} = 10.001$

USE CALC:
 (ALPHA) (WINDOW)
 5: LOGBASE
 $\log_3 10 = 2.096$

Without LOGBASE on calc

$$y = \log_3 10$$

Use
 "Change
 of
 Base"
 formula

$$\frac{\log 10}{\log 3} = 2.0959$$

Properties

$$\boxed{y = \log_a x} \Leftrightarrow \boxed{a^y = x}$$

$$1.) \log_a 1 = 0$$

$$a^0 = 1 \checkmark$$

$$2.) \log_a a = 1$$

$$a^1 = a \checkmark$$

$$3.) \log_a (a^x) = x$$

$$a^x = a^x \checkmark$$

$$4.) \text{If } \log_a x = \log_a y$$

$$\text{then } x = y$$

$$\log_2 (2^3) = 3$$

Reminder #3:

$$2^1 = 2$$

$$3^1 = 3$$

$$2^2 = 4$$

$$3^2 = 9$$

$$2^3 = 8$$

$$3^3 = 27$$

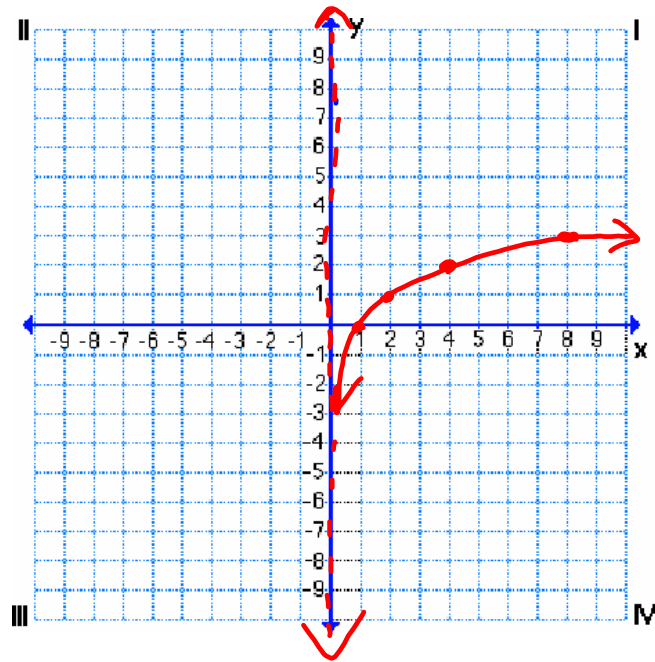
$$2^4 = 16$$

$$3^4 = 81$$

$$2^5 = 32$$

(EX1) Graph $y = \log_2 x$

	x	$y = \log_2 x$	(x, y)
2^0	1	$\log_2 1 = 0$	(1, 0)
2^1	2	$\log_2 2 = 1$	(2, 1)
2^2	4	$\log_2 (2^2) = 2$	(4, 2)



Translation

$$y = \log_a (x - h) + k$$

VA
horizontal
shift

vertical
shift

HW 3.2

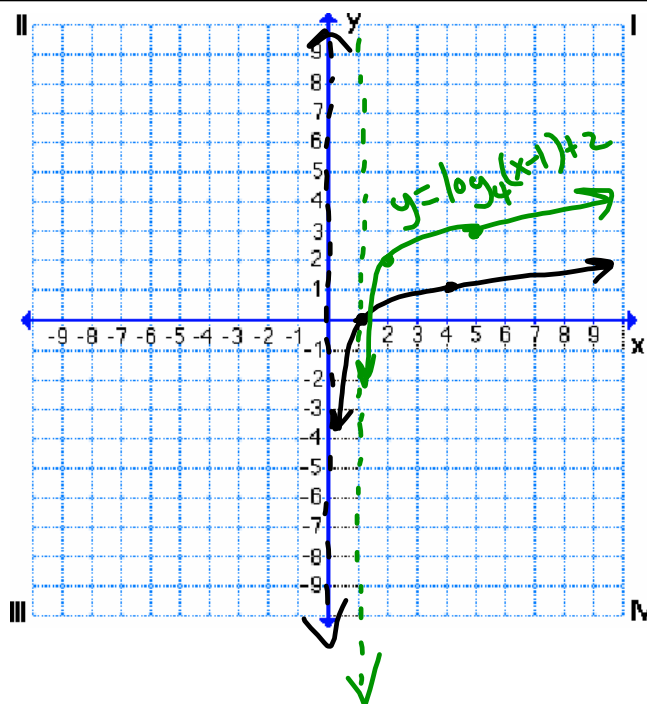
① $y = \log_4(x-1) + 2$

right 1
up 2

$y = \log_4 x$
PARENT

VA

x	$y = \log_4 x$	(x, y)
1	$\log_4 1 = 0$	(1, 0)
4	$\log_4 4 = 1$	(4, 1)
16	$\log_4 16 = 2$	(16, 2)

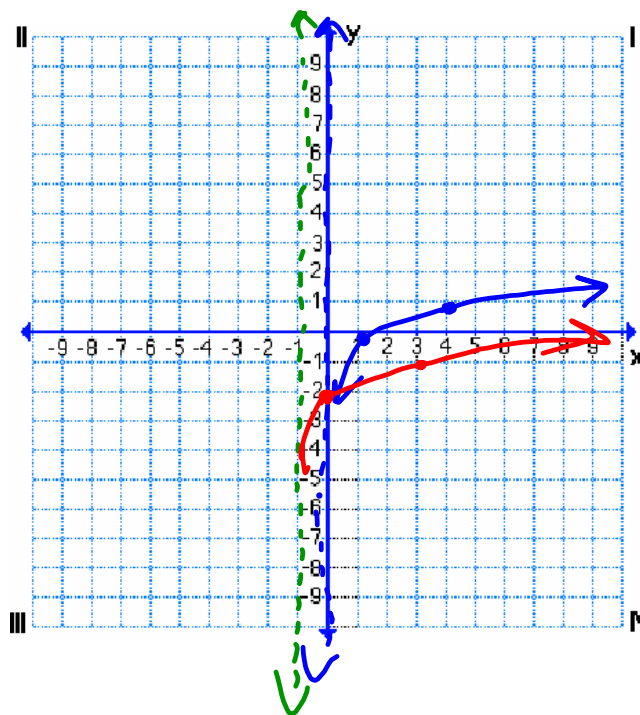
Practice

Graph $y = \log_4(x+1) - 2$

$y = \log_4 x$

left 1
down 2

x	$y = \log_4 x$
1	$\log_4 1 = 0$
4	$\log_4 4 = 1$
16	$\log_4 16 = 2$



NATURAL BASE

$$e \approx 2.7182818 \dots$$

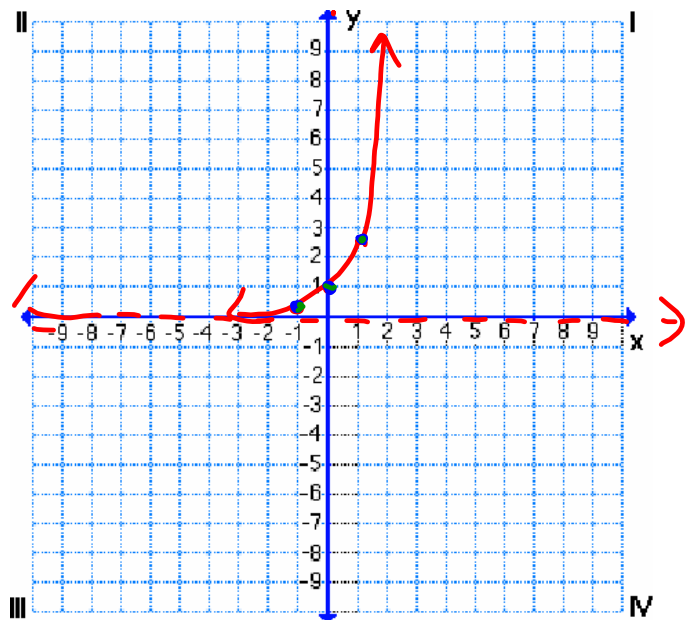
"exponential function with base e"

$$f(x) = e^x$$

Graph

$$y = e^x$$

x	y = e ^x
-1	e ⁻¹ = 1/e ≈ 1/2.7
0	e ⁰ = 1
1	e ¹ = 2.7



NATURAL LOGARITHM

$$y = \ln x \Rightarrow y = \log_e x$$

$$\ln 1 = 0$$

$$\log_e 1 = 0$$

$$\ln e = 1$$

$$\log_e e = 1$$

$$\ln e^x = x$$

$$\log_e e^x = x$$